TECHNOLOGY MERGERS AND ACQUISITIONS IN THE PRESENCE OF AN INSTALLED BASE: A STRATEGIC ANALYSIS

Online Supplement

In all analysis we assume a consumer will buy the new product if she is indifferent between buying it and buying the old product. We use I to denote the incumbent, E the entrant, N the new product, O the old product, H the high type consumers, and L the low type consumers. We use backward induction to derive all equilibrium outcomes.

A. Equilibrium outcomes under competition

Because H owns O and consumers do not repurchase the same products, although the planning horizon has n periods, the vendors can sell their products in at most three stages (we define a stage as a period in which at least one product is sold). If the products are sold in three different stages, then in each stage at most one product can be sold – either I sells O to L, E sells N to L, or E sells N to H. Any other combinations, such as E selling N to both H and L, would reduce the number of stages to fewer than three. Let x denote the period in which the first stage decision is made, y the period for the second stage, and z the period for the third stage, $x < y < z \leq n$.

Suppose we arrive at stage 3. E will sell N to L because L will not buy O if she owns N. Hence, I selling O to L cannot be the equilibrium outcome in stage 3. Similarly, owing to the competition from I and $v_H > v_L$, any price of N that attracts L will also attract H, meaning E cannot sell N only to L before stage 3. This implies E selling N only to H cannot be the equilibrium outcome either.

Accordingly, if we arrive at stage 3, E will sell N to L. The price of N,

$$p_z^N = v_L[U(q_N, n - z + 1) - U(q_O, n - z)]. \quad \text{(A1)}$$

E's profit,

$$\pi_z^E(z : N \to L) = d_L p_z^N = d_L v_L[U(q_N, n - z + 1) - U(q_O, n - z)]. \quad \text{(A2)}$$

With assumption 2, i.e., $\delta q_N > q_O$, $\frac{d\pi_z^E}{dz} < 0$ and $\frac{d^2\pi_z^E}{dz^2} < 0$. Hence, the optimal period to sell N to L, z, is the lower bound, $z^* = y + 1$.

To derive the outcomes in stage 2, we need to list the options of the vendors in stage 1. In stage 1, I's and E's decisions follow the normal form game below.

		Entrant: N → H	N → L	N → H, L	Not sell
Incumbent	O → L	H: N, L: O	H: O, L: ?	H: N, L: ?	H: O, L: O
	Not sell	H: N, L: –	H: O, L: N	H: N, L: N	n.a.

Note that both I and E not selling any products contradicts the supposition that we are in stage 1 (defined only if a product is sold). Hence, we do not consider this combination.

Optimal decisions in stage 2 (period y)

1. H:N, L:O. Only E can sell N to L. Its profit,

$$\pi_y^E(N \to L) = d_L p_y^N = d_L v_L[U(q_N, n - y + 1) - U(q_O, n - y)]. \quad \text{(A3)}$$

When $\delta q_N > q_O$, $\frac{d\pi_y^E}{dy} < 0$ and $\frac{d^2\pi_y^E}{dy^2} < 0$. Hence, the optimal period to sell N to L is the lower bound, $y^* = x+1$. No more products will be sold in the future. Stage 3 does not exist.

2. H:O, L:?. There are two possibilities:

   (a) L owns O. I will not be able to sell any product. E may sell N to L, to H, or to both H and L. It is straightforward to see that selling N only to L is not optimal. If E wants to sell N only to H, it has to ensure H prefers to buy it in stage 2 instead of not buying or buying it in stage 3. This implies

$$\begin{aligned} v_H[U(q_N, n-y+1) - U(q_O, n-y)] - p_y^N \geq \\ \delta^{(z-y)} v_H[U(q_N, n-z+1) - U(q_O, n-z)] - \delta^{(z-y)} p_z^N > 0. \end{aligned} \tag{A4}$$

   By (A4) and the analysis of stage 3, viz., (A1),

$$p_y^N = v_H[U(q_N, 1) - U(q_O, 1)] + \delta v_L[U(q_N, n-y) - U(q_O, n-y-1)]. \tag{A5}$$

   E's profit would then be

$$\begin{aligned} \pi_y^E(y: N \to H; y+1: N \to L) &= d_H p_y^N + \delta d_L p_z^N \\ &= d_H v_H[U(q_N, 1) - U(q_O, 1)] + \delta v_L[U(q_N, n-y) - U(q_O, n-y-1)]. \end{aligned} \tag{A6}$$

   If E wants to sell N to both H and L, it must charge a lower price to attract L, giving profit

$$\pi_y^E(y: N \to H, L) = (d_H + d_L) p_y^N = v_L[U(q_N, n-y+1) - U(q_O, n-y)]. \tag{A7}$$

   Because $d_H v_L^H > 1$, $\pi_y^E(y: N \to H; y+1: N \to L) > \pi_y^E(y: N \to H, L)$, meaning E will sell N only to H in stage 2. The optimal period is the lower bound, $y^* = x+1$.

   (b) L owns N. Once again, I will not be able to sell any product. E will sell N only to H with profit

$$\pi_y^N(y: N \to H) = d_H p_y^N = d_H v_H[U(q_N, n-y+1) - U(q_O, n-y)]. \tag{A8}$$

   The optimal period is the lower bound, $y^* = x+1$.

3. H:N, L:?. There are two possibilities:

   (a) L owns O. The outcome is identical to that in case 1.

   (b) L owns N. All consumers have the best product. Stages 2 and 3 do not exist.

4. H:O, L:O. The outcome is identical to that in case 2(a). E will sell N only to H.

5. H:N, L: –. I and E would compete to sell to L. Although L does not own any product, because of zero marginal cost and the presence of I who will always compete to sell O if E over-charges for N, the situation faced by E is similar to that in case 1, as if L owns O. Hence, the outcome is identical to that in case 1. I will not be able to sell any product.

6. H:O, L: N. The outcome is identical to that in case 2(b).

7. H:N, L: N. Both E and I cannot sell any product. Stages 2 and 3 do not exist.

Optimal decisions in stage 1 (period x)

The above analysis shows that I always earns zero profit in stage 2. The strategy combination, $\{I$: not sell, E: N $\to$ H$\}$, cannot constitute an equilibrium in stage 1 because when E is not selling to L, I can sell O to L at a sufficiently low price and earn a positive profit. Similarly, selling N only to L cannot be the equilibrium strategy in stage 1. With zero marginal cost, the highest price that E can charge for N is $p_x^N = v_L[U(q_N, n-x+1) - U(q_O, n-x)] < v_H[U(q_N, n-x+1) - U(q_O, n-x)]$, which is what H is willing to pay for N. Hence, E will not be able to sell N to L alone, meaning any strategy pairs with E: N $\to$ L cannot constitute an equilibrium in stage 1.

Now, consider the strategy pair $\{I: O \to L,\ E: N \to H\}$, which leads to case 1 in stage 2. L will buy N in stage 2. To sell O in stage 1, I must set the price such that L prefers to buy O in stage 1 instead of not buying, waiting until stage 2 to buy N, or buying N in stage 1. This implies

$$v_L U(q_O, n-x) - p_x^O > \max\{0, \delta v_L U(q_N, n-y+1) - \delta p_y^N, v_L U(q_N, n-x+1) - p_x^N\}. \quad \text{(A9)}$$

Similarly, to sell N to H in stage 1, E must set the price such that H prefers to buy N in stage 1 instead of not buying or waiting until stage 2 to buy N. This implies

$$v_H[U(q_N, n-x+1) - U(q_O, n-x)] - p_x^N \geq \max\{0, \delta v_H[U(q_N, n-y+1) - U(q_O, n-y)] - \delta p_y^N\}. \quad \text{(A10)}$$

Substituting p_y^N from (A3), (A10) implies

$$p_x^N = v_H[U(q_N, 1) - U(q_O, 1)] + \delta v_L[U(q_N, n-x) - U(q_O, n-x-1)]. \quad \text{(A11)}$$

Substituting from (A11), (A9) implies

$$p_x^O = \begin{cases} (v_H - v_L)[U(q_N, 1) - U(q_O, 1)] & \text{if } v_L^H \leq \Upsilon_1, \\ v_L U(q_O, 1) & \text{if } v_L^H > \Upsilon_1, \end{cases} \quad \text{(A12)}$$

where

$$\boxed{\Upsilon_1 \equiv \frac{U(q_N, 1)}{U(q_N, 1) - U(q_O, 1)}}. \quad \text{(A13)}$$

The corresponding profits of I and E are

$$\pi_x^I(x: O \to L) = \begin{cases} d_L(v_H - v_L)[U(q_N, 1) - U(q_O, 1)] & \text{if } v_L^H \leq \Upsilon_1, \\ d_L v_L U(q_O, 1) & \text{if } v_L^H > \Upsilon_1, \end{cases} \quad \text{(A14)}$$

and

$$\pi_x^E(x: N \to H; x+1: N \to L) = d_H v_H[U(q_N, 1) - U(q_O, 1)] + \delta v_L[U(q_N, n-x) - U(q_O, n-x-1)]. \quad \text{(A15)}$$

The optimal period is the lower bound, $x^* = 1$.

Next, suppose E wants to sell N to both H and L. Because of the competition from I, E must charge a sufficiently low price for N to attract L. Hence, its profit is bounded by $v_L[U(q_N, n-x+1) - U(q_O, n-x)] < d_H v_H[U(q_N, 1) - U(q_O, 1)] + \delta v_L[U(q_N, n-x) - U(q_O, n-x-1)]$, which, by (A15), is what E can earn by selling N only to H in stage 1 and then to L in stage 2. Hence, E selling N to both H and L in stage 1 cannot be an equilibrium strategy.

Finally, consider the strategy pair $\{I$: O $\to$ L, E: not sell$\}$, which leads to case 4 and in turn case 2(a) in stage 2. By (A6), E's discounted profit in stage 1,

$$\begin{aligned} &\pi_x^E(x: -; x+1: N \to H; x+2: N \to L) \\ &\quad = \delta d_H v_H[U(q_N, 1) - U(q_O, 1)] + \delta^2 v_L[U(q_N, n-x-1) - U(q_O, n-x-2)]. \end{aligned} \quad \text{(A16)}$$

Comparing with (A15), it is obvious that E prefers to sell N to H in stage 1. Hence, $\{I$: O $\to$ L, E: not sell$\}$ cannot constitute an equilibrium either.

In summary, there exists a **unique equilibrium** in the competition between I and E. In this equilibrium, I will sell L to O and E will sell N to H in period 1, and then E will sell N to L in period 2. The prices are given in (A3), (A11), and (A12) with $x = 1$ and $y = 2$. The following results summarize the equilibrium outcomes.

$$\pi^I(1: O \to L) = \begin{cases} d_L(v_H - v_L)[U(q_N, 1) - U(q_O, 1)] & \text{if } v_L^H \leq \Upsilon_1, \\ d_L v_L U(q_O, 1) & \text{if } v_L^H > \Upsilon_1, \end{cases} \quad \text{(A17)}$$

and

$$\pi^E(1:N \to H; 2:N \to L) = d_H v_H[U(q_N,1) - U(q_O,1)] + \delta v_L[U(q_N,n-1) - U(q_O,n-2)]. \quad \text{(A18)}$$

Now, suppose N is introduced in period $t_0 > 1$. As we explain in Section 3, I will sell O to H in period 0 and to L in period 1. So, when N arrives, E will face a fully-saturated market whereby all consumers own O. The analysis is similar to the analysis of case 2(a) in stage 2. E will sell N to H in period t_0 and to L in period $t_0 + 1$. By (A1) and (A5),

$$p^N_{t_0} = v_H[U(q_N,1) - U(q_O,1)] + \delta v_L[U(q_N,n-t_0) - U(q_O,n-t_0-1)], \quad \text{(A19)}$$

$$p^N_{t_0+1} = v_L[U(q_N,n-t_0) - U(q_O,n-t_0-1)]. \quad \text{(A20)}$$

E's profit in period t_0,

$$\begin{aligned}&\pi^E(t_0:N \to H; t_0+1:N \to L)\\&\quad = d_H v_H[U(q_N,1) - U(q_O,1)] + \delta v_L[U(q_N,n-t_0) - U(q_O,n-t_0-1)].\end{aligned} \quad \text{(A21)}$$

With $\delta q_N > q_O$, E prefers to sell N as soon as it arrives, i.e., in period t_0.

B. Equilibrium outcomes with acquisition and no upgrade policy

The analysis of stage 3 is identical to that in the case with competition, so (A1) and (A2) apply. In stage 1, the vendor has five choices as shown in the following table.

O → L	N → L	N → H	N → H, O → L	N → H, L
H: O, L: O	H: O, L: N	H: N, L: –	H: N, L: O	H: N, L: N

Optimal decisions in stage 2 (period y)

1. H:O, L: O. The consideration is identical to that in case 2(a) in stage 2 in the case with competition. Accordingly, by (A6) and (A7), the vendor will sell N only to H in stage 2 (and then sell N to L in stage 3). Its prices and profit are identical to those in (A1), (A5) and (A6), with $z^* = y + 1$ and $y^* = x + 1$.
2. H:O, L:N. The vendor will sell N only to H. Its price and profit are identical to those in (A8).
3. H:N, L: –. The vendor can either sell N to L in stage 2 or sell O to L in stage 2 followed by selling N to L in stage 3. If it sells N to L in stage 2, its profit is

$$\pi_y(y : N \to L) = d_L p_y^N = d_L v_L U(q_N, n - y + 1). \tag{B1}$$

   If it wants to sell O to L in stage 2, it has to ensure L prefers to buy O in stage 2 instead of not buying or buying N in stage 3. This implies

$$v_L U(q_O, n - y) - p_y^O \geq \delta^{(z-y)} v_L U(q_N, n - z + 1) - \delta^{(z-y)} p_z^N \geq 0. \tag{B2}$$

   Substituting from (A1), (B2) implies

$$p_y^O = v_L U(q_O, 1). \tag{B3}$$

   The vendor's profit,

$$\begin{aligned} \pi_y(y : O \to L; y + 1 : N \to L) &= d_L p_y^O + \delta d_L p_z^N \\ &= d_L v_L U(q_O, 1) + \delta d_L v_L [U(q_N, n - y) - U(q_O, n - y - 1)]. \end{aligned} \tag{B4}$$

   Comparing (B1) with (B4), $\pi_y(y : N \to L) > \pi_y(y : O \to L; y + 1 : N \to L)$. Hence, the vendor will sell N to L in period y. The optimal period is the lower bound, $y^* = x + 1$.
4. H:N, L: O. The vendor will sell N only to L. Its price and profit are identical to those in (A3).
5. H:N, L: N. The vendor will not be able to sell any product. Stage 2 does not exist.

Optimal decisions in stage 1 (period x)

1. Sell O to L. This strategy leads to case 1 in stage 2. The vendor must set the price such that L prefers to buy O in stage 1 instead of not buying or buying N in stage 2 or stage 3. This implies

$$\begin{aligned} &v_L U(q_O, n - x) - p_x^O \\ &\quad \geq \max\{0, \delta^{(y-x)} v_L U(q_N, n - y + 1) - \delta^{(y-x)} p_y^N, \delta^{(z-x)} v_L U(q_N, n - z + 1) - \delta^{(z-x)} p_z^N\}. \end{aligned} \tag{B5}$$

   Substituting from (A1) and (A5), (B5) implies

$$p_x^O = \begin{cases} v_L U(q_O, 1) + \delta(v_H - v_L)[U(q_N, 1) - U(q_O, 1)] & \text{if } v_L^H \leq \Upsilon_1, \\ v_L U(q_O, 2) & \text{if } v_L^H > \Upsilon_1. \end{cases} \tag{B6}$$

   The corresponding profit,

$$\begin{aligned} &\pi(x : O \to L; x + 1 : N \to H; x + 2 : N \to L) \\ &= \begin{cases} d_L v_L U(q_O,1) + \delta(v_H - d_L v_L)[U(q_N, 1) - U(q_O, 1)] \\ \qquad + \delta^2 v_L [U(q_N, n - x - 1) - U(q_O, n - x - 2)] & \text{if } v_L^H \leq \Upsilon_1, \\ d_L v_L U(q_O,2) + \delta d_H v_H [U(q_N, 1) - U(q_O, 1)] \\ \qquad + \delta^2 v_L [U(q_N, n - x - 1) - U(q_O, n - x - 2)] & \text{if } v_L^H > \Upsilon_1. \end{cases} \end{aligned} \tag{B7}$$

The optimal period is the lower bound, $x^* = 1$.

2. Sell N to L. This strategy leads to case 2 in stage 2. The vendor must set the price such that L prefers to buy N in stage 1 instead of not buying or buying it in stage 2, and H prefers to buy N in stage 2. These imply

$$v_L U(q_N, n-x+1) - p_x^N \geq \max\{0, \delta^{(y-x)} v_L U(q_N, n-y+1) - \delta^{(y-x)} p_y^N\}, \tag{B8}$$

and

$$v_H[U(q_N, n-x+1) - U(q_O, n-x)] - p_x^N < 0. \tag{B9}$$

Substituting from (A8), (B8) implies when $v_L^H > \Upsilon_3(x)$, the optimal price of N in stage 1 is $p_x^N = v_L U(q_N, n-x+1)$, which satisfies (B9) if and only if $v_L^H \leq \Upsilon_2(x)$, where

$$\Upsilon_2(x) \equiv \frac{U(q_N, n-x+1)}{U(q_N, n-x+1) - U(q_O, n-x)}, \text{ and } \Upsilon_3(x) \equiv \frac{U(q_N, n-x)}{U(q_N, n-x) - U(q_O, n-x-1)}, \tag{B10}$$

and $\Upsilon_1 > \Upsilon_2(x) > \Upsilon_3(x)$ for all x. Similarly, when $v_L^H \leq \Upsilon_3(x)$, the optimal price of N in stage 1 is $p_x^N = v_L U(q_N, 1) + \delta v_H[U(q_N, n-x) - U(q_O, n-x-1)]$, which always satisfies (B9) because $v_L^H \leq \Upsilon_3(x) < \Upsilon_1$. Taken together, the price of N in stage 1,

$$p_x^N = \begin{cases} v_L U(q_N, 1) + \delta v_H[U(q_N, n-x) - U(q_O, n-x-1)] & \text{if } v_L^H \leq \Upsilon_3(x), \\ v_L U(q_N, n-x+1) & \text{if } \Upsilon_3(x) < v_L^H \leq \Upsilon_2(x), \\ N.A. & \text{if } v_L^H > \Upsilon_2(x). \end{cases} \tag{B11}$$

The corresponding profit,

$$\begin{aligned} &\pi(x: N \to L; x+1: N \to H) \\ &= \begin{cases} d_L v_L U(q_N, 1) \\ \quad + \delta v_H[U(q_N, n-x) - U(q_O, n-x-1)] & \text{if } v_L^H \leq \Upsilon_3(x), \\ d_L v_L U(q_N, n-x+1) \\ \quad + \delta d_H v_H[U(q_N, n-x) - U(q_O, n-x-1)] & \text{if } \Upsilon_3(x) < v_L^H \leq \Upsilon_2(x). \\ N.A. & \text{if } v_L^H > \Upsilon_2(x). \end{cases} \end{aligned} \tag{B12}$$

The optimal period is the lower bound, $x^* = 1$.

3. Sell N to H. This strategy leads to case 3 in stage 2. The vendor must set the price such that H prefers to buy N in stage 1 instead of not buying or buying it in stage 2, and L prefers to buy N in stage 2. These imply

$$\begin{aligned} &v_H[U(q_N, n-x+1) - U(q_O, n-x)] - p_x^N \\ &\qquad \geq \max\{0, \delta^{(y-x)} v_H[U(q_N, n-y+1) - U(q_O, n-y)] - \delta^{(y-x)} p_y^N\}, \end{aligned} \tag{B13}$$

and

$$v_L U(q_N, n-x+1) - p_x^N < 0. \tag{B14}$$

Substituting from (B1), (B13) implies when $v_L^H \leq \Upsilon_3(x)$, the optimal price of N in stage 1 is $p_x^N = v_H[U(q_N, n-x+1) - U(q_O, n-x)]$, which satisfies (B14) if and only if $v_L^H > \Upsilon_2(x)$. This is, however, a contradiction because $\Upsilon_2(x) > \Upsilon_3(x)$. On the other hand, when $v_L^H > \Upsilon_3(x)$, the optimal price is

$$p_x^N = v_H[U(q_N, 1) - U(q_O, 1)] + \delta v_L U(q_N, n-x), \tag{B15}$$

which satisfies (B14) if and only if $v_L^H > \Upsilon_1$. Accordingly, the vendor's profit,

$$\begin{aligned} &\pi(x: N \to H; x+1: N \to L) \\ &= \begin{cases} N.A. & \text{if } v_L^H \leq \Upsilon_1, \\ d_H v_H[U(q_N, 1) - U(q_O, 1)] + \delta v_L U(q_N, n-x) & \text{if } v_L^H > \Upsilon_1. \end{cases} \end{aligned} \tag{B16}$$

The optimal period is the lower bound, $x^* = 1$.

4. Sell N to H and O to L. This strategy leads to case 4 in stage 2. The vendor must set the price such

that H prefers to buy N in stage 1 instead of not buying or buying it in stage 2, and L prefers to buy O in stage 1 instead of not buying or buying N in stage 1 or stage 2. These imply

$$v_H[U(q_N, n-x+1) - U(q_O, n-x)] - p_x^N \\ \geq \max\{0, \delta^{(y-x)} v_H[U(q_N, n-y+1) - U(q_O, n-y)] - \delta^{(y-x)} p_y^N\}, \tag{B17}$$

and

$$v_L U(q_O, n-x) - p_x^O \\ \geq \max\{0, v_L U(q_N, n-x+1) - p_x^N, \delta^{(y-x)} v_L U(q_N, n-y+1) - \delta^{(y-x)} p_y^N\}. \tag{B18}$$

Substituting from (A3), (B17) implies

$$p_x^N = v_H[U(q_N, 1) - U(q_O, 1)] + \delta v_L[U(q_N, n-x) - U(q_O, n-x-1)]. \tag{B19}$$

Similarly, substituting from (A3) and (B19), (B18) implies

$$p_x^O = \begin{cases} (v_H - v_L)[U(q_N, 1) - U(q_O, 1)] & \text{if } v_L^H \leq \Upsilon_1, \\ v_L U(q_O, 1) & \text{if } v_L^H > \Upsilon_1. \end{cases} \tag{B20}$$

The corresponding profit,

$$\pi(x: N \to H, O \to L; x+1: N \to L) \\ = \begin{cases} (v_H - d_L v_L)[U(q_N, 1) - U(q_O, 1)] \\ \quad + \delta v_L[U(q_N, n-x) - U(q_O, n-x-1)] & \text{if } v_L^H \leq \Upsilon_1, \\ d_L v_L U(q_O, 1) + d_H v_H[U(q_N, 1) - U(q_O, 1)] \\ \quad + \delta v_L[U(q_N, n-x) - U(q_O, n-x-1)] & \text{if } v_L^H > \Upsilon_1. \end{cases} \tag{B21}$$

The optimal period is the lower bound, $x^* = 1$.

5. Sell N to H and L. This strategy leads to case 5 in stage 2, i.e., no consumer will buy any item in the future. The vendor must set the price such that both H and L prefer to buy N in stage 1 instead of not buying. This implies $v_H[U(q_N, n-x+1) - U(q_O, n-x)] - p_x^N \geq 0$ and $v_L U(q_N, n-x+1) - p_x^N \geq 0$. Hence, the price of N,

$$p_x^N = \begin{cases} v_H[U(q_N, n-x+1) - U(q_O, n-x)] & \text{if } v_L^H \leq \Upsilon_2(x), \\ v_L U(q_N, n-x+1) & \text{if } v_L^H > \Upsilon_2(x). \end{cases} \tag{B22}$$

The corresponding profit,

$$\pi(x: N \to H, L) = \begin{cases} v_H[U(q_N, n-x+1) - U(q_O, n-x)] & \text{if } v_L^H \leq \Upsilon_2(x), \\ v_L U(q_N, n-x+1) & \text{if } v_L^H > \Upsilon_2(x). \end{cases} \tag{B23}$$

The optimal period is the lower bound, $x^* = 1$.

Equilibrium strategies

The above analysis suggests the optimal period to start selling a product, $x^* = 1$. Accordingly, $\Upsilon_2(x)$ and $\Upsilon_3(x)$ can be simplified to:

$$\boxed{\Upsilon_2 \equiv \frac{U(q_N, n)}{U(q_N, n) - U(q_O, n-1)}}, \text{ and } \boxed{\Upsilon_3 \equiv \frac{U(q_N, n-1)}{U(q_N, n-1) - U(q_O, n-2)}}, \tag{B24}$$

where $\Upsilon_1 > \Upsilon_2 > \Upsilon_3$. To derive the equilibrium strategies, we consider the following ranges of v_L^H:

- When $v_L^H \leq \Upsilon_3$: By (B7) and (B12), $\pi(1: O \to L; 2: N \to H; 3: N \to L) < \pi(1: N \to L; 2: N \to H)$. By (B21) and (B23), $\pi(1: N \to H, O \to L; 2: N \to L) < \pi(1: N \to H, L)$. By (B12) and (B23), $\pi(1: N \to H, L) \geq \pi(1: N \to L; 2: N \to H)$ if and only if

$$v_L^H \geq d_L \Upsilon_1. \tag{B25}$$

- When $\Upsilon_3 < v_L^H \leq \Upsilon_2$: By (B21) and (B23), $\pi(1 : N \to H, O \to L; 2 : N \to L) < \pi(1 : N \to H, L)$. Because $d_H v_L^H > 1$, by (B7) and (B12), $\pi(1 : N \to L; 2 : N \to H) - \pi(1 : O \to L; 2 : N \to H; 3 : N \to L)$ has the sign of $\delta d_L v_L U(q_N, n-1) - \delta d_L v_H [U(q_N, 1) - U(q_O, 1)] > 0$ when $v_L^H \leq \Upsilon_1$. By (B12) and (B23), $\pi(1 : N \to H, L) \geq \pi(1 : N \to L; 2 : N \to H)$ if and only if

$$v_L^H \geq \frac{\Upsilon_2}{1 + \left(\frac{1-d_L}{d_L} \cdot \frac{U(q_N,1)-U(q_O,1)}{U(q_N,n)-U(q_O,n-1)}\right)}. \tag{B26}$$

- When $\Upsilon_2 < v_L^H \leq \Upsilon_1$: By (B21) and (B23), $\pi(1 : N \to H, L) - \pi(1 : N \to H, O \to L; 2 : N \to L) = v_L[U(q_N, 1) - U(q_O, 1)](\Upsilon_1 - v_L^H + d_L) + \delta v_L U(q_O, n-2) > 0$ because $v_L^H \leq \Upsilon_1$. By (B7) and (B23), $\pi(1 : N \to H, L) - \pi(1 : O \to L; 2 : N \to H; 3 : N \to L)$ has the sign of

$$\begin{aligned} &\frac{1+\delta}{\delta} + \frac{(1-d_L)U(q_O,1) + \delta U(q_O, n-2)}{\delta[U(q_N,1) - U(q_O,1)]} + d_L - v_L^H \\ &\quad = \frac{1}{\delta} + \frac{(1-d_L)U(q_O,1)}{\delta[U(q_N,1) - U(q_O,1)]} + d_L + \frac{U(q_N,1) - U(q_O,1) + U(q_O,n-2)}{U(q_N,1) - U(q_O,1)} - v_L^H \\ &\quad > \frac{1}{\delta} + \frac{(1-d_L)U(q_O,1)}{\delta[U(q_N,1) - U(q_O,1)]} + d_L + \Upsilon_1 - v_L^H > 0, \end{aligned}$$

because $U(q_O, n-2) \geq U(q_O, 1)$ when $n \geq 3$, and $v_L^H \leq \Upsilon_1$.

- When $v_L^H > \Upsilon_1$: By (B7) and (B16), $\pi(1 : O \to L; 2 : N \to H; 3 : N \to L) > \pi(1 : N \to H; 2 : N \to L)$ if and only if

$$v_L^H < \frac{d_L U(q_O,2) - \delta U(q_N,1) - \delta^2 U(q_O, n-3)}{(1-\delta)(1-d_L)[U(q_N,1) - U(q_O,1)]}. \tag{B27}$$

By (B7) and (B21), $\pi(1 : O \to L; 2 : N \to H; 3 : N \to L) > \pi(1 : N \to H, O \to L; 2 : N \to L)$ if and only if

$$v_L^H < \frac{\delta(1+d_L)U(q_O,1) - \delta U(q_N,1)}{(1-\delta)(1-d_L)[U(q_N,1) - U(q_O,1)]}. \tag{B28}$$

By (B7) and (B23), $\pi(1 : O \to L; 2 : N \to H; 3 : N \to L) > \pi(1 : N \to H, L)$ if and only if

$$v_L^H > \frac{U(q_N,2) - d_L U(q_O,2) + \delta^2 U(q_O, n-3)}{\delta(1-d_L)[U(q_N,1) - U(q_O,1)]}. \tag{B29}$$

By (B16) and (B21), $\pi(1 : N \to H; 2 : N \to L) > \pi(1 : N \to H, O \to L; 2 : N \to L)$ if and only if

$$d_L < \frac{\delta U(q_O, n-2)}{U(q_O,1)}. \tag{B30}$$

By (B16) and (B23), $\pi(1 : N \to H; 2 : N \to L) > \pi(1 : N \to H, L)$ if and only if

$$v_L^H > \frac{\Upsilon_1}{1-d_L}. \tag{B31}$$

By (B21) and (B23), $\pi(1 : N \to H, O \to L; 2 : N \to L) > \pi(1 : N \to H, L)$ if and only if

$$v_L^H > \Upsilon_1 + \frac{d_L}{1-d_L} + \frac{\delta U(q_O, n-2)}{(1-d_L)[U(q_N,1) - U(q_O,1)]}. \tag{B32}$$

Taken together, (B27) to (B32) define the parameters for each of the four candidate strategies to be optimal in equilibrium. For (B27) and (B29) to hold, we must have

$$d_L > \frac{U(q_N,1) + \delta^2 U(q_O, n-3)}{(1+\delta)U(q_O,1)}, \tag{B33}$$

which is possible only if $n = 3$ because $\delta q_N > q_O$. Hence, for all $n > 3$, strategy $\{1 : O \to L; 2 : N \to$

$H; 3 : N \to L\}$ is dominated. Similarly, for (B28) and (B29) to hold, we must have

$$d_L > \frac{U(q_N, 1) - \delta^{(n-1)} U(q_O, 1)}{U(q_O, 1)}. \tag{B34}$$

By (B27), (B28), (B29), (B33), and (B34), substituting $n = 3$, strategy $\{1 : O \to L; 2 : N \to H; 3 : N \to L\}$ is optimal if and only if

$$\begin{aligned} &\frac{U(q_N, 2) - d_L U(q_O, 2)}{\delta(1 - d_L)[U(q_N, 1) - U(q_O, 1)]} < v_L^H \\ &\quad < \min\left\{\frac{d_L U(q_O, 2) - \delta U(q_N, 1)}{(1 - \delta)(1 - d_L)[U(q_N, 1) - U(q_O, 1)]}, \frac{\delta(1 + d_L) U(q_O, 1) - \delta U(q_N, 1)}{(1 - \delta)(1 - d_L)[U(q_N, 1) - U(q_O, 1)]}\right\} \end{aligned} \tag{B35}$$

and

$$\max\{\frac{1}{(1 + \delta) q_O}, \frac{1}{q_O} - \delta^2\} < d_L < 1. \tag{B36}$$

By (B29), (B31), and (B32), strategy $\{1 : N \to H, L\}$ is optimal if and only if $v_L^H \leq \min\{\frac{\Upsilon_1}{1-d_L}, \Upsilon_1 + \frac{d_L}{1-d_L} + \frac{\delta U(q_O, n-2)}{(1-d_L)[U(q_N,1)-U(q_O,1)]}\}$, and $v_L^H \leq \frac{U(q_N,2)-d_L U(q_O,2)}{\delta(1-d_L)[U(q_N,1)-U(q_O,1)]}$ when $n = 3$.

By (B27), (B30), and (B31), strategy $\{1 : N \to H; 2 : N \to L\}$ is optimal if and only if $d_L < \frac{\delta U(q_O,n-2)}{U(q_O,1)}$ and $v_L^H > \frac{\Upsilon_1}{1-d_L}$, and $v_L^H \geq \frac{d_L U(q_O,2)-\delta U(q_N,1)}{(1-\delta)(1-d_L)[U(q_N,1)-U(q_O,1)]}$ when $n = 3$.

By (B28), (B30), and (B32), strategy $\{1 : N \to H, O \to L; 2 : N \to L\}$ is optimal if and only if $d_L \geq \frac{\delta U(q_O,n-2)}{U(q_O,1)}$, $v_L^H > \Upsilon_1 + \frac{d_L}{1-d_L} + \frac{\delta U(q_O,n-2)}{(1-d_L)[U(q_N,1)-U(q_O,1)]}$, and $v_L^H \geq \frac{\delta(1+d_L)U(q_O,1)-\delta U(q_N,1)}{(1-\delta)(1-d_L)[U(q_N,1)-U(q_O,1)]}$ when $n = 3$.

Additional analysis: If $\delta = 1$, by (B16) and (B21), $\pi(1 : N \to H; 2 : N \to L) - \pi(1 : N \to H, O \to L; 2 : N \to L) = v_L[U(q_O, n-2) - d_L U(q_O, 1)] > 0$ because $U(q_O, n-2) \geq U(q_O, 1)$ when $n \geq 3$, and $d_L < 1$. Hence, strategy $\{1 : N \to H, O \to L; 2 : N \to L\}$ is dominated. Only three strategies will remain in the equilibrium. Then, by (B7) and (B16), $\pi(1 : O \to L; 2 : N \to H; 3 : N \to L) > \pi(1 : N \to H; 2 : N \to L)$ if and only if

$$d_L > \frac{U(q_N, 1) + U(q_O, n-3)}{U(q_O, 2)}, \tag{B37}$$

which replaces (B27) and is identical to (B33) when $\delta = 1$. Here again, (B37) can be satisfied only if $n = 3$. Hence, by (B29) and (B37), when $\delta = 1$, substituting $n = 3$, strategy $\{1 : O \to L; 2 : N \to H; 3 : N \to L\}$ is optimal if and only if

$$v_L^H > \frac{U(q_N, 2) - d_L U(q_O, 2)}{(1 - d_L)[U(q_N, 1) - U(q_O, 1)]} \tag{B38}$$

and

$$\frac{1}{2q_O} < d_L < 1. \tag{B39}$$

Similarly, by (B29) and (B31), strategy $\{1 : N \to H, L\}$ is optimal if and only if $v_L^H \leq \frac{\Upsilon_1}{1-d_L}$ and $v_L^H \leq \frac{U(q_N,2)-d_L U(q_O,2)}{(1-d_L)[U(q_N,1)-U(q_O,1)]}$ when $n = 3$. By (B37) and (B31), strategy $\{1 : N \to H; 2 : N \to L\}$ is optimal if and only if $v_L^H > \frac{\Upsilon_1}{1-d_L}$, and $d_L \leq \frac{1}{2q_O}$ when $n = 3$.

Comparing with the case when $0 < \delta < 1$, the conditions required for strategies $\{1 : N \to H, L\}$, $\{1 : N \to H; 2 : N \to L\}$, $\{1 : O \to L; 2 : N \to H; 3 : N \to L\}$ to constitute the equilibrium are *weaker* because strategy $\{1 : N \to H, O \to L; 2 : N \to L\}$ is now sub-optimal.

The following table summarizes the equilibrium outcomes. In general, as d_L decreases, i.e., as the number of low type consumers decreases relative to the number of high type consumers, the vendor prefers the strategies in the lower rows in the table.

Parametrization	Vendor's strategy	Vendor's profit	Price
$v_L^H < d_L \Upsilon_1$	$1: N \to L$ $2: N \to H$	$d_L v_L U(q_N,1)$ $+\delta v_H[U(q_N,n-1)-U(q_O,n-2)]$	$p_1^N = v_L U(q_N,1)+\delta v_H[U(q_N,n-1)-U(q_O,n-2)]$ $p_2^N = v_H[U(q_N,n-1)-U(q_O,n-2)]$
$d_L \Upsilon_1 \le v_L^H \le \Upsilon_3$	$1: N \to H, L$	$v_H[U(q_N,n)-U(q_O,n-1)]$	$p_1^N = v_H[U(q_N,n)-U(q_O,n-1)]$
$\Upsilon_3 < v_L^H < \frac{\Upsilon_2}{1+\left(\frac{1-d_L}{d_L}\cdot\frac{U(q_N,1)-U(q_O,1)}{U(q_N,n)-U(q_O,n-1)}\right)}$	$1: N \to L$ $2: N \to H$	$d_L v_L U(q_N,n)$ $+\delta d_H v_H[U(q_N,n-1)-U(q_O,n-2)]$	$p_1^N = v_L U(q_N,n)$ $p_2^N = v_H[U(q_N,n-1)-U(q_O,n-2)]$
$\frac{\Upsilon_2}{1+\left(\frac{1-d_L}{d_L}\cdot\frac{U(q_N,1)-U(q_O,1)}{U(q_N,n)-U(q_O,n-1)}\right)} \le v_L^H \le \Upsilon_2$	$1: N \to H, L$	$v_H[U(q_N,n)-U(q_O,n-1)]$	$p_1^N = v_H[U(q_N,n)-U(q_O,n-1)]$
$\Upsilon_2 < v_L^H \le \Upsilon_1$	$1: N \to H, L$	$v_L U(q_N,n)$	$p_1^N = v_L U(q_N,n)$
$v_L^H > \Upsilon_1$	$1: N \to H, L^{(i)}$	$v_L U(q_N,n)$	$p_1^N = v_L U(q_N,n)$
	$1: N \to H^{(ii)}$ $2: N \to L$	$d_H v_H[U(q_N,1)-U(q_O,1)]+\delta v_L U(q_N,n-1)$	$p_1^N = v_H[U(q_N,1)-U(q_O,1)]+\delta v_L U(q_N,n-1)$ $p_2^N = v_L U(q_N,n-1)$
	$1: O \to L^{(iii)}$ $2: N \to H$ $3: N \to L$	$d_L v_L U(q_O,2)+\delta d_H v_H[U(q_N,1)-U(q_O,1)]$ $+\delta^2 v_L[U(q_N,n-2)-U(q_O,n-3)]$	$p_1^O = v_L U(q_O,2)$ $p_2^N = v_H[U(q_N,1)-U(q_O,1)]$ $+\delta v_L[U(q_N,n-2)-U(q_O,n-3)]$ $p_3^N = v_L[U(q_N,n-2)-U(q_O,n-3)]$
	$1: N \to H, O \to L^{(iv)}$ $2: N \to L$	$d_L v_L U(q_O,1)+d_H v_H[U(q_N,1)-U(q_O,1)]$ $+\delta v_L[U(q_N,n-1)-U(q_O,n-2)]$	$p_1^O = v_L U(q_O,1)$ $p_1^N = v_H[U(q_N,1)-U(q_O,1)]$ $+\delta v_L[U(q_N,n-1)-U(q_O,n-2)]$ $p_2^N = v_L[U(q_N,n-1)-U(q_O,n-2)]$

Notes: $\Upsilon_1 \equiv \frac{U(q_N,1)}{U(q_N,1)-U(q_O,1)}$, $\Upsilon_2 \equiv \frac{U(q_N,n)}{U(q_N,n)-U(q_O,n-1)}$, $\Upsilon_3 \equiv \frac{U(q_N,n-1)}{U(q_N,n-1)-U(q_O,n-2)}$; $\Upsilon_1 > \Upsilon_2 > \Upsilon_3$.

$^{(i)}$ Optimal if and only if $v_L^H \le \min\{\frac{\Upsilon_1}{1-d_L}, \Upsilon_1 + \frac{d_L}{1-d_L} + \frac{\delta U(q_O,n-2)}{(1-d_L)[U(q_N,1)-U(q_O,1)]}\}$, and $v_L^H \le \frac{U(q_N,2)-d_L U(q_O,2)}{\delta(1-d_L)[U(q_N,1)-U(q_O,1)]}$ when $n=3$.

$^{(ii)}$ Optimal if and only if $d_L < \frac{\delta U(q_O,n-2)}{U(q_O,1)}$ and $v_L^H > \frac{\Upsilon_1}{1-d_L}$, and $v_L^H \ge \frac{d_L U(q_O,2)-\delta U(q_N,1)}{(1-\delta)(1-d_L)[U(q_N,1)-U(q_O,1)]}$ when $n=3$.

$^{(iii)}$ Optimal if and only if $n=3$, $\frac{U(q_N,2)-d_L U(q_O,2)}{\delta(1-d_L)[U(q_N,1)-U(q_O,1)]} < v_L^H < \min\left\{\frac{d_L U(q_O,2)-\delta U(q_N,1)}{(1-\delta)(1-d_L)[U(q_N,1)-U(q_O,1)]}, \frac{\delta(1+d_L)U(q_O,1)-\delta U(q_N,1)}{(1-\delta)(1-d_L)[U(q_N,1)-U(q_O,1)]}\right\}$, and $\max\{\frac{1}{(1+\delta)q_O}, \frac{1}{q_O}-\delta^2\} < d_L < 1$.

$^{(iv)}$ Optimal if and only if $0<\delta<1$, $d_L \ge \frac{\delta U(q_O,n-2)}{U(q_O,1)}$, $v_L^H > \Upsilon_1 + \frac{d_L}{1-d_L} + \frac{\delta U(q_O,n-2)}{(1-d_L)[U(q_N,1)-U(q_O,1)]}$, and $v_L^H \ge \frac{\delta(1+d_L)U(q_O,1)-\delta U(q_N,1)}{(1-\delta)(1-d_L)[U(q_N,1)-U(q_O,1)]}$ when $n=3$.

C. Equilibrium outcomes with acquisition and upgrade policy

The game sequence and the analysis of stage 3 are identical to those in the case with acquisition and no upgrade policy. Hence, (A1) and (A2) apply.

Optimal decisions in stage 2 (period y)

There are again five cases. Cases 1, 2, 4 and 5 do not involve selling the same product to consumers with different purchase histories in stage 2. Hence, the outcomes in the scenario with no upgrade policy directly apply. The only difference lies in case 3, whereby the vendor can either sell N to L in stage 2 or sell O to L in stage 2 followed by selling N to L in stage 3. With the upgrade policy, if L do not buy O in stage 2, they will not be able to buy N in stage 3 at the price stated in (A1) because that price will be offered only to consumers holding O. This implies

$$v_L U(q_O, n-y) - p_y^O \geq 0 \tag{C1}$$

and

$$p_y^O = v_L U(q_O, n-y). \tag{C2}$$

The vendor's profit,

$$\begin{aligned}\pi_y(y: O \to L; y+1: N \to L) &= d_L p_y^O + \delta d_L p_z^N \\ &= d_L v_L U(q_O, n-y) + \delta d_L v_L[U(q_N, n-y) - U(q_O, n-y-1)].\end{aligned} \tag{C3}$$

By (C3) and (B1), $\pi_y(y: N \to L) > \pi_y(y: O \to L; y+1: N \to L)$. Hence, the vendor prefers to sell N to L in period y. The optimal period is the lower bound, $y^* = x+1$.

Optimal decisions in stage 1 (period x)

1. Sell O to L. This strategy leads to case 1 in stage 2, i.e., the vendor will sell N only to H in stage 2 and then to L in stage 3. With an upgrade policy, however, the vendor can now prevent L from leapfrogging to N in stage 2. Hence, the price of O in stage 1 is subject to only one constraint, $v_L U(q_O, n-x) - p_x^O \geq 0$, which implies

$$p_x^O = v_L U(q_O, n-x). \tag{C4}$$

   Together with (A6), the vendor's profit,

$$\begin{aligned}\pi(x: O \to L; x+1: N \to H; x+2: N \to L) &= d_L v_L U(q_O, n-x) \\ &+ \delta d_H v_H[U(q_N, 1) - U(q_O, 1)] + \delta^2 v_L[U(q_N, n-x-1) - U(q_O, n-x-2)].\end{aligned} \tag{C5}$$

   The optimal period is the lower bound, $x^* = 1$.

2. Sell N to L. This strategy leads to case 2 in stage 2. Similar to case 1, because of the upgrade policy, if L does not buy N in stage 1, it will not be able to buy it in stage 2 at the discounted price. The vendor must set the price such that H prefers to buy N in stage 2 instead of 1, i.e., $v_L U(q_N, n-x+1) - p_x^N \geq 0$ and $v_H[U(q_N, n-x+1) - U(q_O, n-x)] - p_x^N < 0$. These two constraints are satisfied if and only if $v_L^H \leq \Upsilon_2(x)$. Hence,

$$p_x^N = \begin{cases} v_L U(q_N, n-x+1) & \text{if } v_L^H \leq \Upsilon_2(x), \\ N.A. & \text{if } v_L^H > \Upsilon_2(x). \end{cases} \tag{C6}$$

   The corresponding profit,

$$\begin{aligned}&\pi(x: N \to L; x+1: N \to H) \\ &= \begin{cases} d_L v_L U(q_N, n-x+1) \\ \qquad + \delta d_H v_H[U(q_N, n-x) - U(q_O, n-x-1)] & \text{if } v_L^H \leq \Upsilon_2(x), \\ N.A. & \text{if } v_L^H > \Upsilon_2(x). \end{cases}\end{aligned} \tag{C7}$$

   The optimal period is the lower bound, $x^* = 1$.

3. Sell N to H. This strategy leads to case 3 in stage 2. With an upgrade policy, L cannot buy N in stage 1. Hence, the vendor only needs to set an upgrade price such that H prefers to buy N in stage

1 instead of not buying or buying it in stage 2. This implies

$$\begin{aligned} v_H[U(q_N, n-x+1) - U(q_O, n-x)] - p_x^U \\ \geq \max\{0, \delta^{(y-x)} v_H[U(q_N, n-y+1) - U(q_O, n-y)] - \delta^{(y-x)} p_y^N\}, \end{aligned} \tag{C8}$$

Substituting from (B1), (C8) implies

$$p_x^U = \begin{cases} v_H[U(q_N, n-x+1) - U(q_O, n-x)] & \text{if } v_L^H \leq \Upsilon_3(x), \\ v_H[U(q_N, 1) - U(q_O, 1)] + \delta v_L U(q_N, n-x) & \text{if } v_L^H > \Upsilon_3(x). \end{cases} \tag{C9}$$

The corresponding profit,

$$\begin{aligned} &\pi(x : N \to H; x+1 : N \to L) \\ &= \begin{cases} d_H v_H[U(q_N, n-x+1) - U(q_O, n-x)] + \delta d_L v_L U(q_N, n-x) & \text{if } v_L^H \leq \Upsilon_3(x), \\ d_H v_H[U(q_N, 1) - U(q_O, 1)] + \delta v_L U(q_N, n-x) & \text{if } v_L^H > \Upsilon_3(x). \end{cases} \end{aligned} \tag{C10}$$

The optimal period is the lower bound, $x^* = 1$.

4. Sell N to H and O to L. This strategy leads to case 4 in stage 2. With an upgrade policy, L cannot buy N in stage 2 without buying O in stage 1. They cannot buy N in stage 1 either because it is offered only at the upgrade price. The vendor must set the price such that H prefers to buy N in stage 1 instead of not buying or buying it in stage 2. The upgrade price must satisfy the constraint

$$\begin{aligned} v_H[U(q_N, n-x+1) - U(q_O, n-x)] - p_x^U \\ \geq \max\{0, \delta^{(y-x)} v_H[U(q_N, n-y+1) - U(q_O, n-y)] - \delta^{(y-x)} p_y^N\}. \end{aligned} \tag{C11}$$

Substituting from (A3), (C11) implies

$$p_x^U = v_H[U(q_N, 1) - U(q_O, 1)] + \delta v_L[U(q_N, n-x) - U(q_O, n-x-1)]. \tag{C12}$$

Similarly, the vendor must set the price of O such that $v_L U(q_O, n-x) - p_x^O \geq 0$. Hence, the price of O is given by (C4). Taken together, the vendor's profit,

$$\begin{aligned} \pi(x : N \to H, O \to L; x+1 : N \to L) = d_H v_H[U(q_N, 1) - U(q_O, 1)] \\ + \delta v_L[U(q_N, n-x) - U(q_O, n-x-1)] + d_L v_L U(q_O, n-x). \end{aligned} \tag{C13}$$

The optimal period is the lower bound, $x^* = 1$.

5. Sell N to H and L. This strategy leads to case 5 in stage 2. Following similar analysis as case 5 in Section B,

$$\begin{cases} p_x^U = v_H[U(q_N, n-x+1) - U(q_O, n-x)], \quad p_x^N = v_L U(q_N, n-x+1) & \text{if } v_L^H \leq \Upsilon_2(x), \\ p_x^U = p_x^N = v_L U(q_N, n-x+1) & \text{if } v_L^H > \Upsilon_2(x). \end{cases} \tag{C14}$$

The corresponding profit,

$$\pi(x : N \to H, L) = \begin{cases} d_H v_H[U(q_N, n-x+1) - U(q_O, n-x)] \\ \qquad + d_L v_L U(q_N, n-x+1) & \text{if } v_L^H \leq \Upsilon_2(x), \\ v_L U(q_N, n-x+1) & \text{if } v_L^H > \Upsilon_2(x). \end{cases} \tag{C15}$$

The optimal period is the lower bound, $x^* = 1$.

Equilibrium strategies

By (C5) and (C13), $\pi(1 : N \to H, O \to L; 2 : N \to L) > \pi(1 : O \to L; 2 : N \to H; 3 : N \to L)$, meaning strategy $\{1 : O \to L; 2 : N \to H; 3 : N \to L\}$ is always dominated. Further, by (C7), (C10), (C13), and (C15), for all $v_L^H \leq \Upsilon_2$, strategy $\{1 : N \to H, L\}$ dominates all other strategies.

We next compare the vendor's strategies in cases 3 to 5 when $v_L^H > \Upsilon_2$. By (C10), (C13), and

(C15), $\pi(1:N\to H,L)\geq\pi(1:N\to H;2:N\to L)$ if and only if

$$v_L^H \leq \frac{\Upsilon_1}{1-d_L}. \tag{C16}$$

$\pi(1:N\to H,L)\geq\pi(1:N\to H,O\to L;2:N\to L)$ if and only if

$$v_L^H \leq \Upsilon_1 + \frac{d_L}{1-d_L} + \frac{\delta U(q_O,n-2)}{U(q_N,1)-U(q_O,1)}. \tag{C17}$$

$\pi(1:N\to H;2:N\to L) > \pi(1:N\to H,O\to L;2:N\to L)$ if and only if

$$d_L < \frac{\delta U(q_O,n-2)}{U(q_O,n-1)}. \tag{C18}$$

The following table summarizes the equilibrium outcomes in different parametrization. Note that the equilibrium outcomes with upgrade policy are robust when $\delta = 1$.

Parametrization	Vendor's strategy	Vendor's profit	Price
$v_L^H \leq \Upsilon_2$	$1: N \to H, L$	$d_H v_H [U(q_N, n) - U(q_O, n-1)] + d_L v_L U(q_N, n)$	$p_1^U = v_H [U(q_N, n) - U(q_O, n-1)]$ $p_1^N = v_L U(q_N, n)$
$v_L^H > \Upsilon_2$.	$1: N \to H, L^{(i)}$	$v_L U(q_N, n)$	$p_1^N = v_L U(q_N, n)$
	$1: N \to H^{(ii)}$ $2: N \to L$	$d_H v_H [U(q_N, 1) - U(q_O, 1)] + \delta v_L U(q_N, n-1)$	$p_1^U = v_H [U(q_N, 1) - U(q_O, 1)] + \delta v_L U(q_N, n-1)$ $p_2^N = v_L U(q_N, n-1)$
	$1: N \to H, O \to L^{(iii)}$ $2: N \to L$	$d_H v_H [U(q_N, 1) - U(q_O, 1)]$ $+\delta v_L [U(q_N, n-1) - U(q_O, n-2)]$ $+d_L v_L U(q_O, n-1)$	$p_1^O = v_L U(q_O, n-1)$ $p_1^U = v_H [U(q_N, 1) - U(q_O, 1)]$ $+\delta v_L [U(q_N, n-1) - U(q_O, n-2)]$ $p_2^N = v_L [U(q_N, n-1) - U(q_O, n-2)]$

Note: $\Upsilon_1 \equiv \frac{U(q_N,1)}{U(q_N,1)-U(q_O,1)}$, $\Upsilon_2 \equiv \frac{U(q_N,n)}{U(q_N,n)-U(q_O,n-1)}$, $\Upsilon_3 \equiv \frac{U(q_N,n-1)}{U(q_N,n-1)-U(q_O,n-2)}$; $\Upsilon_1 > \Upsilon_2 > \Upsilon_3$.

$^{(i)}$ Optimal if and only if $v_L^H \leq \min\{\frac{\Upsilon_1}{1-d_L}, \Upsilon_1 + \frac{d_L}{1-d_L} + \frac{\delta U(q_O,n-2)}{U(q_N,1)-U(q_O,1)}\}$.

$^{(ii)}$ Optimal if and only if $v_L^H > \frac{\Upsilon_1}{1-d_L}$ and $d_L < \frac{\delta U(q_O,n-2)}{U(q_O,n-1)}$.

$^{(iii)}$ Optimal if and only if $v_L^H > \Upsilon_1 + \frac{d_L}{1-d_L} + \frac{\delta U(q_O,n-2)}{U(q_N,1)-U(q_O,1)}$ and $d_L \geq \frac{\delta U(q_O,n-2)}{U(q_O,n-1)}$.